

The Empire Open Math Contest

Question 1. $\triangle ABC$ is a right triangle with $A = (2, 3)$, $B = (5, 7)$, and $\angle B = 90^\circ$. Given that the y -coordinate of C is $-\frac{1}{5}$, find the x -coordinate of C , expressed as a fraction in lowest terms. This fraction is written as $\frac{p}{q}$, where p, q are positive integers and the greatest common factor between them is 1. Report your answer as $p + q$.

Question 2. You arrive at Penn Station and step outside and notice a peculiar smell. You look up and notice that an analog clock reads 4:20. What is the smaller angle, in degrees, between the hour hand and the minute hand?

Question 3. The council of Pigeons convenes in Central Park to decide their next target. There are 12 pigeon-lords who each control different sections of the city. Before starting the meeting every pigeon-lord dabs up (shakes hands) with every other pigeon-lord. How many dabs (handshakes) occur?

Question 4. You're riding the 6 train and encounter a delay. To your fortunate surprise you actually have signal but to your dismay every time you refresh your phone on Google Maps to see your estimated time of arrival at Grand Central the time goes up...

The sequence of times is given by T_n as follows:

$$T_1 = 3$$

and

$$T_{n+1} = 2T_n + 1.$$

What is T_5 ?

Question 5. A bag contains 3 blue balls and 5 orange balls (Go Knicks!). Two balls are drawn without replacement (meaning after the ball is taken out of the bag, another copy of that ball miraculously appears inside the bag). The probability that they are the same color is

$$\frac{a}{b},$$

where a and b are relatively prime positive integers. What is $a + b$?

Question 6. A hot dog costs \$2 in Central Park and a pretzel costs \$3. Suppose your net worth is \$37. How many ways can spend your entire net worth in Central Park on hot dogs and pretzels?

Question 7. You always forget which street your friend lives on in Manhattan, but you do remember it's below 100th Street. You call to ask, but your friend is annoyed and will only tell you that that the street number leaves a remainder of 2 when divided by 5 and a remainder of 3 when divided by 7. What is the sum of all possible street numbers?

Question 8. Governor Hochul has assigned you to be the head of the New York State Lottery. It has been decided that the answer to this year's lottery is a 3 digit number. While deciding what the number should be, Mr. Crabs walks out of the TV and lets you know that he insists that the lottery should be rigged. He desires that the number ends with a 5 and also has to be divisible by 9. How many possible lottery numbers are there that can meet Governor Hochul and Mr. Crabs's constraints?

Question 9. The year is 2090 and the term “rizz” is cool again. These days however kids use telepathic brain computer interfaces to keep the meme cool they don’t even articulate what magic rizz-number is. You have been tasked with finding the number and have just received the following information from discord: $s(n)$ be the sum of the digits of the two-digit number n . If

$$n + s(n) = 61,$$

what is n ?

Question 10. Your realtor has helped have find an excellent deal on a rectangular studio rental listing. Unfortunately the listing has no photos, and only states that the perimeter is 50 feet and the area is 150 square feet. What is the length of the longer side of the studio?

Question 11. Bailey has a Brownstone in Brooklyn. She’s looking at her view but unfortunately some developers have put some eyesore skyerscrapers right in the way. Luckily on her block the skyscrapers are only on points (a, b) where $ab = 12$. We want to count how many skyscrapers she can see from her house so we are asking:

How many ordered pairs (a, b) of positive integers with $1 \leq a, b \leq 30$ have the property that ab is divisible by 12?

Question 12. The Manhattan street grid forms perfect blocks, each 250 feet long and 900 feet wide. A pigeon flies in a straight line from the bottom-right corner of one block to the top left corner of the block above it. The distance the pigeon flew can be expressed as $a\sqrt{b}$, where a and b are positive integers and b is not divisible by the square of any prime. What is $a + b$?

Question 13. The number 24 is an abundant number. Its sum of divisors is bigger than it. since we like abundant numbers, let us now define a “chill” number as a number whose sum of divisors is an abundant number. Consider the 24-“chill” numbers: these are the numbers whose sum of all positive divisors is exactly 24. Find the sum of all 24-“chill” numbers.

Question 14. Eh, forget it, theres no way to yassify this one. Gonna just ask it straight: How many integers less than 2027 are relatively prime to it?

Question 15. (Viktor Krapivin, NYCMath) Ramanujan shows up suddenly in NYC in 2026 and presents you with the mysterious infinite continued fraction

$$x = 3 + \frac{1}{6 + \frac{1}{6 + \frac{1}{6 + \ddots}}}$$

x can be expressed as \sqrt{k} . Find k .