The Empire Open Math Contest 2 - June 2026 - Competitive Division

Question 1. The State of Liberty and you decide to go to the Hard Rock Cafe in Times Square to unwind on a Friday Night. Lady Liberty pulls out her phone to connect to the wifi but unfortunately its blocked. You ask the waiter and they point you to a sign. It's a bit dark so you rub your eyes to see clearly and in gothic chalk on a black background the following was inscribed:

Find the sum of all integers x satisfying two properties:

$$|2x - 7| < 15$$

and the integer remainder of the x after dividing by 8 is 1 or more technically put:

$$x^2 \equiv 1 \pmod 8.$$

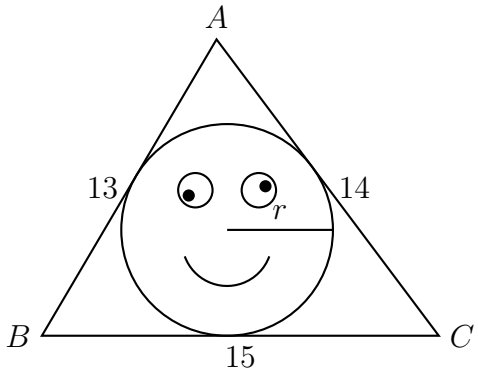
Question 2. (By Richard Liu, NYCMath) There are 15 subway stations across New York City. Some of them are connected by subway lines, which are operated by three divisions: the IRT, the IND, and the BMT. It is known that even if any one division suspends all of its service, it is still possible to travel between any two stations using the remaining two divisions (possibly with transfers). What is the smallest number of subway connections in the city?

Question 3. (By Viktor Krapivin, NYCMath) Finance Fred buys his 5th Frolex watch. Tbh, he can't even read the time on it. Its totally messed up with like 221 hour markings around a circle. In an attempt to culture himself Fred decides to learn about alternative systems of time. Inspired by this he riddles you the following problem: Find the smallest positive integer x such that x^7 divided by 221 has the same remainder as 2026 divided by 221. Or put more technically, find the smallest positive integer x such that

$$x^7 \equiv 2026 \mod 221$$

Question 4. The price of Bigcoin is a chart that never goes down and takes on integer values each day for 4 days (so you can represent the chart as a 4 digit positive integer). The SEC would like your help to compute all the possible charts that Bigcoin can take on. Said more concretely, how many four-digit positive integers have digits in nondecreasing order from left to right?

Question 5. In the diagram below (not drawn to scale), A circle is tangent to all three sides of the triangle. The side lengths of the triangle are 13, 14, and 15. What is the radius of the circle?



Question 6. Divisibility Dan has been summoned from his secret base under the G train. He riddles you the following: how many subsets of $\{1, 2, 3, \ldots, 10\}$ have sum divisible by 5? (note the empty set is a valid subset and its sum is 0)

Question 7. Jay-Z and Euclid were having some difficulties with their time machine and got teleported to Chelsea right during the Knicks playoffs. They wanted to get to Grand Central and there is no Taxi that is willing to drive them so they mus walk. Euclid is claustrophobic and so would prefer to avoid Madison Square Garden. Help them by computing how many lattice paths there from $(0, 0)$ to $(7, 5)$, using only steps right and up, do not pass through $(3, 2)$.

Question 8. Optimus Prime visit's Delmonico's to Order some Prime Rib. The staff at Delmonico's tell Optimus that he can have free refills if he can answer this question about Number Theory. How many positive integers $n \leq 1000$ have exactly 12 positive divisors and are not divisible by 5?

Question 9. (By Terence Coelho, NYCMath) Squidward and Patrick are on a game show, where there are 10 doors labeled 1, 2, 3, ... 10. Exactly one of these doors contains $\$10,000$ dollars, and this door is chosen uniformly at random. When the game starts, Squidward and Patrick will each secretly choose a door number. If either of them chose the correct door, the game will end and the player who chose the door with the prize will take home the $\$10,000$ (sharing it between them if they both chose the correct door). Otherwise, they will continue, not learning what door the other just chose.

Squidward knows Patrick extremely well (in particular he's not very bright) and is certain that Patrick will choose doors 1, 2, 3, etc. in that order until the game ends. If Squidward chooses to exploit this and maximize his expected reward from the game, what would his expected reward be?

Question 10. Isaac Newton himself (this looks like a hint if you don't need a hint) has a hard math problem. Let r and s be the two roots of

$$x^2 - 6x + 1 = 0.$$

Find

$$r^4 + s^4.$$

Question 11. One day you suddenly wake up in the matrix... except its unfortunately the Matrix Classic which features 0.1p low res graphics. You look around you and the world is a grid of glowing orbs upon a dark empty void of a background. At every integer coordinate is an orb. You're trying to figure out where you are "supposed" to be and so you start by measuring your room's volume. To do this you will count the orbs that you see which happens to be equivalent to the following problem:

How many ordered triples (a, b, c) of integers satisfy

$$a + b + c = 0$$

and

$$-5 \leq a, b, c \leq 5?$$

Question 12. Aliens descend upon NYC and decide to park their ship directly above the Astor Place Cube in the East village. A giant laser shoots down and suddenly the cube starts to glow! For each edge of the cube the laser creates a glowing circle with the edge as the diameter. The circle is also perpendicular to the line from the center of the cube to the midpoint of the edge. In this way the aliens constructed 12 circles. At how many points do at least two circles intersect?

Question 13. Sally and the crew were hanging out at Domino park playing Dominoes. A giant soccer ball suddenly hit their game and scattered all the dominoes! The crew was trying to remember their exact configuration but have gotten stuck. The game was played on a 2×10 rectangle with normal 2 units by 1 unit dominoes. Sally did remember that the rectangle was perfectly tiled/covered by non overlapping dominoes and there were exactly 2 vertical dominoes. Can you help her count how many possible configurations could have been in place?

Question 14. Frodo Baggins is in the town of Rivendell and feeling a little lazy. Instead of walking he has decided to book a flight on Spirit Airlines to the Shire and is trying to pack as much treasure as he can. His approved suitcase can store a total of 1 cubic meter. He has received from his Fellowship infinitely many objects, in fact for every natural number n he has received exactly 3 objects of size $1/n$ cubic meters. He can't pack it all and he wants to waste no space so he needs your help to know: how many distinct ordered triples (a, b, c) of positive integers exactly satisfy

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1?$$

(Fun fact, Frodo's treasure also reveals the volume of Rivendell's Treasure Vault is infinite but that is not a mystery we will concern ourselves on today's math contest).

Question 15. (Proposed by Patrick Chen (Google), originally proposed by Elliot Line in Actually Good Math Problems on Facebook) Bobby and the friends decide to order a pepperoni pizza pie from Empire Slice and Co. When their order is ready the Chef runs in yelling "We have a problem!!!"

The Chef explains: "I thought I was baking a regular pizza but when I pulled it out the pepperoni had formed a face and the Pizza came to life! It said "Go Spurs!" and I was terrified so I accidentally chopped it and now the slice is all wrong. I'll tell you what. If you can guess the product of the areas to the left and right of the slice I made I'll give you the pizza for free. The pizza is a perfect regular nonagon with area 9, the slice in the middle has area 2 and contains exactly one entire side of the nonagon. Let x be the area of the pizza to the left of the slice, and let y be the area of the pizza to the right of the slice (so that $x + y + 2 = 9$. Compute the product xy.

Bobby and the group stare down at the pizza and now look to you for some help. Please tell them the right answer.

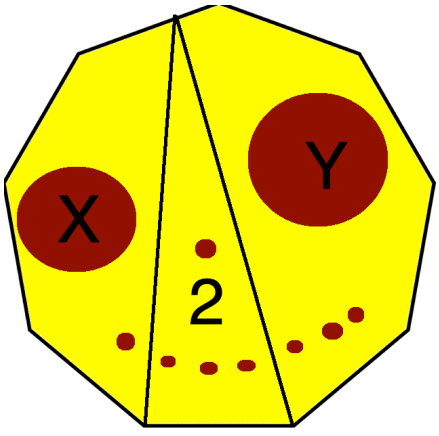


FIGURE 1. The magical pizza of total area 9